THE DYNAMIC LOAD FACTOR OF PRESSURE VESSELS IN DEFLAGRATION EVENTS

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ABSTRACT

There are vessels that could be subjected to rapidly rising pressure load during deflagration events. The dynamic load factor (DLF) which accounts for dynamic load effects in such events can be used to bridge the gap between transient and static pressure loads. This paper addresses the issues in estimating the DLF for a deflagration event. A unified methodology for estimating the DLF is first presented. The methodology is then validated with existing references. Key parameters determining DLFs are identified during the validation. Case studies are used to illustrate how to use the methodology. Finally, this paper is concluded with the findings and a suggested procedure for estimating DLFs in deflagration events.

INTRODUCTION

DuPont plants operate many low to high pressure vessels that may be subjected to dynamic pressure loads due to occasional deflagration events [1]. Total time for pressure rise and decay in such events can last from tens of milliseconds to a few seconds. In such cases, estimating DLFs, which accounts for dynamic load effects and can be used to calculate equivalent static pressure load, is essential for both design and fitness-for-service assessments of these vessels.

ASME BPVC Section VIII Division 3 [2] provides some guidance on estimating dynamic load effects for detonation events in Code case 2564 [3], which occur much faster than deflagration events, but provides no guidance on how to evaluate dynamic load effects in deflagration events. ASME BPVC Section VIII Division 1 Appendix H [4] provides design guidance for deflagration events by referring to NFPA-69 [5] and NFPA-68 [6]. Similar guidance can also be found in ASME BPVC Section VIII Division 2 Annex 4.D [7]. NFPA-68 [6] suggests that without a detailed structural dynamic analysis, a DLF of 1.5 should be used to cover the dynamic load effects in a venting deflagration event. NFPA-68 [6] provides no guidance on how to estimate such a DLF, but indicates that using a DLF of 1.5 is conservative.

Literature review showed that detonation events instead of deflagration events were the focus of research involving dynamic pressure loads in pressure vessels or piping systems. For example, Duffey, Rodriguez and Romero [8] highlighted the difference between the transient impulsive and quasi-static pressure loading in a vessel under high explosive detonation events; Leishear [9] studied the hoop stresses caused by a shock wave front in a gas-filled cylinder, and found that the critical velocity of the shock in the cylinder plays an important role in relating transient to static hoop stresses; Yip and Haroldsen [10] applied ASME BPVC Section VIII Division 3 Code Case 2564 [3] to life assessment of the stainless steel explosive destruction system (EDS) vessels. These works provide insight to understanding dynamic load effects in deflagration events.

To meet the need of accurately estimating the DLF in a deflagration event, a unified methodology has been developed at DuPont. The unified methodology simplifies the pressurized vessel as a single degree-of-freedom (DOF) mass-spring system, assumes a trapezoid shape pressure-time curve in deflagration events, and eventually relates the DLF with the natural frequency of the vessel breathing mode and the deflagration pressure-time curve in an analytical solution. The methodology has been validated and applied to various vessels experiencing deflagration events. Studies revealed that the ratio of the pressure rise time to the natural period of the vessel breathing mode is the controlling parameter determining DLF.

This paper summarizes the work done at DuPont: the development of the unified methodology for estimating the DLF in a deflagration event; validation of the developed methodology; findings; case studies; and a suggested procedure for accurately estimating DLFs.
DLF FOR AN ARBITRARY TRAPEZOID EXCITATION

To derive an analytical solution for estimating DLFs in deflagration events, the following assumptions were made:
1. The vessel is simplified as a single DOF mass-spring system shown in Figure 1. Its characteristic parameter is the period of the vessel breathing mode, $T_n$.
2. The deflagration pressure-time curve is assumed to be a trapezoid as shown in Figure 2a. The curve can be divided into four regimes: rise, step, decay and residual zones.
3. The derivation of the DLF is limited to single DOF linear elastic vibration response. This implies that plasticity, strain rate dependence, and wave propagation are beyond the scope of the present study.

For a given non-dimensional excitation, a typical non-dimensional transient response is shown in Figure 3. The DLF equals the ratio of the system peak transient response, $X_{\text{max}}$, to the system response at the quasi-static load $F_0$, i.e. $X_0$. In other words, DLF is the non-dimensional peak response shown in Figure 3.

Similar to the simulated pressure-time curve for a venting deflagration event [1], Figure 4 highlights the load variation in the event and differentiates the dynamic load from the static load. The operating pressure load before the deflagration event should be treated as quasi-static load. However, the rapid pressure rise and decay in a venting
The system transient response can be derived using the convolution integral [11]
\[ x(t) = \int_{0}^{t} F(\tau) g(t-\tau) d\tau, \quad (1) \]

where \( F(\tau) \) is the excitation load defined in Figure 2a,
\[ g(t) = \frac{1}{m\omega_n} \sin(\omega_n t), \quad (2) \]
is the system response to a unit impulse, and \( \omega_n \) is the system circular frequency.

Using Equations (1) and (2), the non-dimensional transient response can be derived as piece-wise functions in different time regimes: rise, step, decay and residual zones. It should be noted that the transient time response in residual zone is the combined results of loads in rise, step, and decay zones. After tedious mathematical manipulation, the non-dimensional transient time response can be expressed as

\[
\frac{X}{X_0} = \begin{cases} 
0 & t < 0 \\
1 & 0 \leq t < t_0 \\
\frac{\sin(2\pi t \beta)}{2\pi \beta} & t_0 \leq t < t_1 \\
\frac{1}{2\pi \beta} - \frac{\sin(2\pi t \beta)}{2\pi \beta} & t_1 \leq t < t_2 \\
\frac{\sin(2\pi (t_2 - t_1) \beta)}{2\pi \beta} & t_2 \leq t < t_3 \\
\frac{\sin(2\pi (t_1 - t_3) \beta)}{2\pi \beta} & t_3 \leq t < t_4 \\
\frac{\sin(2\pi (t_4 - t_3) \beta)}{2\pi \beta} & t_4 \leq t < t_5 \\
\frac{\sin(2\pi (t_5 - t_4) \beta)}{2\pi \beta} & t_5 \leq t < t_6 \\
\frac{\sin(2\pi (t_6 - t_5) \beta)}{2\pi \beta} & t_6 \leq t < t_7 \\
\frac{\sin(2\pi (t_7 - t_6) \beta)}{2\pi \beta} & t_7 \leq t < \infty.
\end{cases}
\]

Compared to Equation (3), the final analytical solution of DLF is even more complicated so that a computer program is needed for computing DLFs.

With the developed computer program, the non-dimensional transient time response, such as the one shown in Figure 3, can be easily obtained for an arbitrary excitation shown in Figure 2. DLF can then be determined with a known non-dimensional transient time response.

**RESPONSE SPECTRUM AND VALIDATION**

Based on the analytical solution of the transient time response to an arbitrary excitation presented in the previous section, the response spectrum or shock spectrum, in which the system peak response is expressed as a function of the system natural frequency [11], was further derived. The complexity involving in the response spectrum is that the peak response could occur in any one of the rise, step, decay or residual zones because of the time lag between the excitation and response [12]. Similar time lag was also found in finite element simulation of cylinder shells subjected to water hammer dynamic loads [13].

Equation (3) gives the transient time response in terms of displacement to a trapezoid excitation for each loading zone shown in Figure 2. Differentiating the displacement with respect to time yields the velocity. The maximum response occurs at zero velocity points or start/end points of the time regime [12]. Multiple zero velocity points may exist for the transient time response in a time zone. The largest value of the values at these zero velocity points and start/end points must be used to construct the system peak response. With this scheme, the non-dimensional peak response to a trapezoid excitation, i.e. DLF, can be analytically expressed as a function of the system natural frequency.

The derived response spectrum was then validated by three special cases with existing analytical solutions. First, the excitation is a step impulse with a constant slope front shown in Figure 5a.

The general loading case shown in Figure 2b can be reduced to this special case with
\[ t_1 = \infty, t_2 = 0, \beta = t_0 = \frac{T_0}{T_n}. \]

Accordingly, the general spectrum is reduced to
\[ \frac{X_{\text{max}}}{X_0} = 1 + \frac{\sin(\pi \beta)}{\pi \beta}. \quad (4) \]

The peak response given in Equation (4) is identical to that derived in Reference [11]. The peak response is plotted in Figure 5b as a function of \( \beta \), the ratio of the load rise time to the system natural period.
As illustrated in Figure 5b, $\beta$ is the controlling parameter for determining DLF. As $\beta$ approaches 0, DLF becomes as large as 2. As $\beta$ is greater than 5, DLF is less than 1.058.

The pressure-time curve in a non-venting deflagration event may be simplified as the curve shown in Figure 5a. As shown in Figure 5b, the DLF in such an event varies depending on the ratio of the pressure rise time to the system natural period, i.e., $\beta$. As $\beta$ is less than 1, DLF could be as large as 2. As $\beta$ is greater than 5, dynamic load effects become negligible.

In the second case, representing the step zone, the excitation is a rectangular impulse. The excitation and response spectrum for this case are shown in Figure 6.

The general loading case shown in Figure 2b can be reduced to this special case with $t_0 = 0, t_2 = 0, \beta = t_1 = \frac{T_1}{T_n}$. Special treatments, taking limits at zero, are needed to enforce $t_0 = 0$ and $t_2 = 0$ in Equation (3). For example, the response in the rise zone can be rewritten as $\frac{2\pi t - \sin(2\pi t)}{2\pi t_0}$. Using Taylor’s series and noting $0 \leq t \leq t_0$, the expression can be further simplified as

$$
\lim_{t_0 \to 0} \left[ \frac{2\pi t - (2\pi t) + \frac{(2\pi t)^3}{3!} + \ldots}{2\pi t_0} \right] = \lim_{t_0 \to 0} \left[ \frac{2\pi t}{2\pi t_0} \left[ \frac{(2\pi t)^2}{3!} + \ldots \right] \right] = 0.
$$

The response in the step zone can be written as $1 - \frac{\sin(\pi t_0)}{\pi t_0} \cos[2\pi(t - \frac{t_0}{2})]$. Taking limits further reduces the expression to
\[
1 - \lim_{t_0 \to 0} \frac{\sin(\pi t_0)}{\pi t_0} \cdot \lim_{t_0 \to 0} \cos \left[ 2\pi \left( 1 - \frac{t_0}{2} \right) \right] = 1 - \cos(2\pi t).
\]

Using similar techniques, the responses in the decay and residual zones can also be reduced. The general spectrum is then reduced to

\[
X_{\text{max}} = \begin{cases} \frac{2\sin(\pi \beta)}{2} & \beta < 0.5 \\ \pi \beta & \beta \geq 0.5 \end{cases}.
\]

(5)

The peak response given in Equation (5) was also derived in References [8, 14]. As indicated in Figure 6b, DLF remains at a value of 2 as long as the ratio of the load period to the system natural period is greater than 0.5.

A symmetric triangular impulse, representing the rise and decay zones, is the third case for the validation. The general loading case shown in Figure 2b can be reduced to this special case with \( t_0 = t_2, t_1 = 0, \beta = t_0 = \frac{T_0}{T_n} \). The peak response in the residual zone can then be simplified to

\[
X_{\text{max}} = \frac{2 \sin(\pi \beta) \sin(\pi \beta)}{\pi \beta}.
\]

(6)

The same expression was also given in Reference [14].

Figure 7 shows a symmetric triangular impulse excitation and its response spectrum. The dotted line in Figure 7b represents the peak response in the residual zone given in Equation (6). All the other lines represent peak responses in other zones such as step zone and decay zone. To authors’ best knowledge, there is no analytical solution available in the literature for those time regimes. The upper envelope of all these peak responses corresponding to different time zones constitutes the response spectrum.

Two observations can be made from Figure 7b:

1. DLF becomes less than 1.075 as the ratio of the load rise time to the system natural period is greater than 3;
2. The largest value of DLF is about 1.5 which occurs at the ratio of 0.453.

FIGURE 7: (a) EXCITATION – A SYMMETRIC TRIANGULAR IMPULSE; (b) THE RESPONSE SPECTRUM.

Figure 7b helps gain insight to understanding dynamic load effects in a venting deflagration event. The pressure-time curve in such an event may be simplified as the one shown in Figure 7a. Corresponding DLF varies depending on the ratio of the pressure rise time to the system natural period, i.e. \( \beta \). As \( \beta \) is less than 1, DLF could be as large as 1.5, which is coincidentally the same as the DLF suggested by NFPA-68 [6]. As \( \beta \) is greater than 3, dynamic load effects become negligible.

The validation helped build confidence in the derived analytical solution, given in Equation (3), of the transient time response to a trapezoid excitation. More importantly, it helped identify two key parameters in determining DLF: the shape of pressure-time curve; the ratio of the load rise time to the system natural period. As this ratio is greater than 3, dynamic load effects most likely become insignificant.

CASE STUDIES

The first case study is a possible deflagration event in a high pressure vessel. In the deflagration event, the internal pressure rises to 193% of the operating pressure in less than 40
milliseconds and then drops back to the operating pressure within 100 milliseconds.

To estimate DLF for the deflagration event, the system breathing mode natural period is needed. A simplified axisymmetric finite element (FE) model was employed to calculate the natural frequency of the vessel breathing mode. The FE analysis consists of two analysis steps. In the first analysis step, the vessel is pressurized to the operating pressure statically. Frequency extraction (eigenvalue analysis) is then carried out in the subsequent step.

The pressure-time curve in the deflagration event can be normalized as shown in Figure 8.

![Normalized Simplified Pressure-Time Curve](image)

**FIGURE 8: THE NORMALIZED PRESSURE-TIME CURVE IN A POSSIBLE VENTING DEFLAGRATION EVENT IN A HIGH PRESSURE VESSEL.**

The ratio of the load rise time to the system breathing mode natural period is 21.36. Using the computer program described in the previous section, the transient time response can be obtained and plotted as shown in Figure 9.

The DLF in the deflagration event is estimated as 1.009 based on the transient time response shown in Figure 9.

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If operating pressure is denoted as $p_{op}$, then the peak pressure in the deflagration event can be denoted as $1.93p_{op}$. Hence, the dynamic pressure increase is $0.93p_{op}$, and the equivalent static peak pressure in the deflagration event is $1.936p_{op}$, i.e. $p_{op} + 0.93p_{op} \times 1.006 = 1.936p_{op}$. Apparently, the dynamic load effect is negligible in this event.

To further confirm that the dynamic load effect is negligible in the deflagration event, both an explicit dynamic analysis with the actual pressure-time curve and an implicit static analysis were carried out using ABAQUS/Explicit and ABAQUS/Standard, respectively. Theoretically speaking, explicit dynamic analyses can capture all dynamic load effects including wave propagation.

The explicit dynamic analysis showed that the system kinetic energy is negligible compared to the system strain energy. Comparison between the explicit dynamic analysis and the implicit static analysis demonstrated that the difference between two analyses is absolutely negligible.

The second case study is the deflagration event that occurred in a pressure separator [1]. The simulated pressure-time curve is close to a triangle and can be simplified as the curve shown in Figure 10, in which the pressure has been normalized. In the case study, the operating pressure is assumed to be 2,600 psi and the peak pressure in the deflagration event is assumed to be 7,000 psi.

![Simulated Pressure-Time Curve](image)

**FIGURE 9: THE NON-DIMENSIONAL TRANSIENT TIME RESPONSE IN THE DEFLAGRATION EVENT BASED ON THE SINGLE DOF SYSTEM.**

![Normalized Transient Time Response](image)

**FIGURE 10: A SIMULATED PRESSURE-TIME CURVE IN AN OCCURRED DEFLAGRATION EVENT.**
Without knowing the natural frequency of the separator breathing mode, the case study here is intended for illustrating how to estimate the DLF using the newly developed methodology, and the impact of the variation of the natural frequency of the system breathing mode on DLFs.

If the natural frequency of the separator breathing mode is 4 Hz, then the normalized pressure-time curve and non-dimensional transient time response are as plotted in Figure 11.

![Figure 11](image)

FIGURE 11: $T_n = 0.25 \text{Second}, \beta = 5.408$ (a) THE NON-DIMENSIONAL PRESSURE-TIME CURVE; (b) THE NON-DIMENSIONAL TRANSIENT TIME RESPONSE.

The non-dimensional peak response shown in Figure 11b is 1.036, i.e. DLF. The equivalent static peak pressure is 7,158 psi, i.e. $2,600 + 4,400 \times 1.036 = 7,158$.

If the natural frequency of the separator breathing mode is 0.4 Hz, then the normalized pressure-time curve and non-dimensional transient time response are as plotted in Figure 12.

![Figure 12](image)

FIGURE 12: $T_n = 2.5 \text{Second}, \beta = 0.541$ (a) THE NON-DIMENSIONAL PRESSURE-TIME CURVE; (b) THE NON-DIMENSIONAL TRANSIENT TIME RESPONSE.

The non-dimensional peak response shown in Figure 12b is 1.476, i.e. DLF. The equivalent static peak pressure is estimated as 9,094 psi, i.e. $2,600 + 4,400 \times 1.476 = 9,094$.

The response spectrum corresponding to the deflagration pressure-time curve shown in Figure 10 is obtained and plotted in Figure 13.

![Figure 13](image)

FIGURE 13: THE RESPONSE SPECTRUM FOR THE DEFLAGRATION IN THE SEPARATOR.
As found in the previous section, the ratio of the pressure rise time to the natural period of the separator breathing mode is the controlling parameter in determining DLFs in the deflagration event, as illustrated in Figure 13. As the ratio is greater than 4, dynamic load effects become negligible. This observation implies that dynamic load effects can be neglected as long as the natural frequency of the separator breathing mode is greater than 2.96 Hz.

Avoiding significant dynamic load effects should be one design consideration. If a new separator has the same deflagration pressure-time curve as shown in Figure 10, then Figure 13 can be used to guide the design. For example, $\beta > 2$ may be needed for avoiding significant dynamic load effects, which in turn requires that the natural frequency of the new separator breathing mode is greater than 1.48 Hz.

CONCLUSIONS

The DLF in a deflagration event can be estimated using the new methodology presented in this paper with a pressure-time curve and the natural frequency of the vessel breathing mode. The methodology is based on a single DOF linear elastic vibration response of the vessel so that plasticity, strain rate dependence and wave propagation are beyond the scope of the present study.

The ratio of the pressure rise time to the natural period of the vessel breathing mode is identified as the controlling parameter determining DLF. The dynamic load effects for high pressure vessels in venting deflagration events may be negligible because the structure is quite stiff.

A procedure for estimating DLF in a deflagration event is suggested as follows:
1. Use the maximum value without known pressure-time curves: 1.5 for a venting deflagration event, and 2.0 for a non-venting deflagration event.
2. Use the new methodology presented in this paper to estimate DLF more accurately with known pressure-time curves. The natural frequency of the vessel breathing mode can be determined using finite element analysis (FEA).
3. Fully evaluate the dynamic load effects using explicit dynamic FEA.

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REFERENCES


[5] National Fire Protection Association (NFPA) 69, Standard on Explosion Prevention Systems, Chapter 5, Deflagration Pressure Containment, issue effective with the applicable Addenda of the ASME BPVC.

[6] National Fire Protection Association (NFPA) 68, Guide for the Venting of Deflagrations, issue effective with the applicable Addenda of the ASME BPVC.


